The full dimension theory of some complete lattices

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Conference talk, by focusing on the dimension theory of some complete lattices. Let us first recall what sThe goal of this talk is to continue the Szegedhould be meant by "dimension".

For a ring R, let $\mathrm{FP}(R)$ denote the class of finitely generated projective right R-modules. The nonstable K-theory of R is the (commutative) monoid $V(R)$ of all isomorphism classes of objects from $\mathrm{FP}(R)$, with addition defined by $[X] + [Y] = [X \oplus Y]$, where $[X]$ denotes the isomorphism class of X. So $V(R)$ is a precursor of the classical $K_0(R)$. We are interested in the case where R is von Neumann regular, right self-injective.

A related dimension theory, using Murray - von Neumann equivalence, exists for so-called AW*-algebras.

For a lattice L, the dimension monoid $\mathrm{Dim}L$ of L is the commutative monoid defined by generators $D(x, y)$, where $x \leq y$ in L, and relations $D(x, x) = 0$, $D(x, z) = D(x, y) + D(y, z)$ for $x \leq y \leq z$, and $D(x \wedge y, x) = D(y, x \vee y)$, for all x, y, z in L. It turns out that $\mathrm{Dim}L$ is a precursor of the congruence lattice of L. We are interested in the case where L is complete, complemented, modular, and upper continuous.

We present a common abstraction that encompasses all the three contexts encountered above, called "espaliers". We show how to elucidate completely the dimension theory of espaliers, via continuous functions from extremely disconnected Stone spaces to the non-negative reals plus a bunch of infinite cardinals, with applications to the three contexts outlined above. In particular, it turns out that the structure spaces for W*-algebras (von Neumann algebras) and AW*-algebras are not the same.